An Evaluative Calculus Project: Applying Bloom’s Taxonomy to the Calculus Classroom

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Abstract: In education theory, Bloom’s taxonomy is a well-known paradigm to describe domains of learning and levels of competency. In this article I propose a calculus capstone project that is meant to utilize the sixth and arguably the highest level in the cognitive domain, according to Bloom et al.: evaluation. Although one may assume that mathematics is a value-free discipline, and thus the mathematics classroom should be exempt from focusing on the evaluative aspect of higher-level cognitive processing, I surmise that we as mathematics instructors should consider incorporating such components into our courses. The article also includes a brief summary of my observations and a discussion of my experience during the Fall 2008 semester, when I used the project described here in my Calculus I course.

Keywords: Calculus, Bloom’s taxonomy, capstone project, evaluation.

1. INTRODUCTION

“Those who can do. Those who can’t teach. Those who can’t teach, leach teachers.”

Thus goes a newer version of the familiar irreverent adage on teaching by George Bernard Shaw. However, most of us disagree with Shaw, and many among us know that we have a lot to learn from education theory. This really struck home for me when I first learned about Bloom’s taxonomy [4].

As a smug mathematics teacher (see Figure 1) I had always assumed that mathematics, the queen of sciences, provided the perfect medium to teach our students how to think analytically, and thus develop their critical thinking skills. However, as I read about the six levels of the cognitive domain of Bloom’s Taxonomy (knowledge, comprehension, application, analysis, synthesis, and

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evaluation). I had difficulty coming up with examples of college-level mathematics problems that fit the higher levels.

Among those problems I typically assign to my students I could easily find some aiming for the first four out of the six. I found a few examples for the fifth level, too, when I tried. However, I could not find examples for the highest level of cognitive achievement: evaluation.

My first reaction was that this sixth level was incompatible with mathematics. After all, I thought, mathematics is a value-free discipline. That, however, is merely avoiding the issue. If a central purpose of education is to create responsible adults who are capable of critical thinking and decision-making under various circumstances, my assumption that mathematics is essential for a complete education started sounding less convincing. Making value judgments is a part of making good decisions, and it makes sense to include evaluation as one of the goals of education. If we mathematics teachers cannot find ways to address this highest level of the cognitive domain, then our claim for the centrality of mathematics to the intellectual development of our students may seem less justified.

After some initial contemplation of the above, I rolled up my proverbial sleeves and started thinking about the issue more seriously. This article describes the subsequent work that resulted.

In particular, I start the article with the assumption that Bloom’s taxonomy gives us a workable scheme to think about our goals in the mathematics classroom. Given this background, I propose that mathematics teachers should consider incorporating evaluative components into their courses. To this end, I describe a capstone project I developed for and used in a calculus course. I hope that others will test similar ideas in their own classrooms, and share their experiences. Some will perhaps go on to develop evaluative projects of their own.

In the next section, I provide a brief overview of Bloom’s taxonomy and use several simple calculus tasks as examples for the first five levels. Section
Applying Bloom’s Taxonomy to the Classroom

3 focuses on the sixth level of Bloom’s taxonomy, and argues for its relevance to the mathematics classroom. Section 4 introduces the evaluative project mentioned above. In this section I also discuss my experience from when I used this project in my Calculus I course during the Fall 2008 semester. Section 5 wraps up this article.

2. BLOOM’S TAXONOMY AND CALCULUS

During any program of study, learners can display different levels of competency and thinking behavior. First there is the mastery of facts related to the subject studied. In history, for instance, these facts may be certain dates or names of historical figures, in calculus we might think of basic definitions. Then there are several levels of comprehension, from a basic understanding of definitions, to the capacity to compare different concepts, to the ability to apply owned knowledge to new situations. A yet higher level of thinking and learning behavior is the creative formation of new knowledge based on old.

Education researchers have studied learning levels extensively, and proposed several paradigms to describe the objectives of education. The most well-known classification of educational goals was developed in 1956 by a committee chaired by Benjamin Bloom [4].

In this article I uniformly use the descriptive language of Bloom’s Taxonomy. In fact this original language has now been mostly replaced by the modified version of Anderson and Krathwohl [2]. In addition, there are several alternative taxonomies in the literature, see for instance [5] for a taxonomy of educational objectives for the mathematics classroom. Nonetheless, I adhere to the original taxonomy of [4] since it is known and universally accepted across disciplines and national borders.

2.1. Bloom’s Taxonomy: The Cognitive Domain

Bloom’s Taxonomy organizes the goals of education into three domains. The cognitive domain relates to the intellectual part of education that is knowledge-based. The affective domain refers to the attitudinal changes that education can bring about. The psychomotor domain involves the development of mastery in motor skills. Showing a student how to factor a given polynomial focuses on the cognitive domain, while motivating him or her to be open to trying out a new method of solving a differential equation addresses the affective domain. Using a meter stick or blocks to describe the concept of addition engages the psychomotor domain.

In this article I focus on the cognitive domain. Bloom et al. [4] develop a six-tiered scheme to describe educational goals in this domain. These are, in order of increasing sophistication: Knowledge, Comprehension, Application,
In Bloom’s Taxonomy, Knowledge includes knowledge of terminology, specific facts, methods of dealing with these specific facts, and the universals and abstractions of a given field [4, p. 62]. The main cognitive process involved is information retrieval; the kind of information involved may be simply factual or substantially deep.

Comprehension requires the skills needed to translate, interpret, and extrapolate from knowledge [4, pp. 89–90]. It denotes a basic and simplistic level of understanding. Standard examples involve situations where students are expected to rephrase a definition, or summarize a paragraph in their own words.

Application refers to implementing a relevant technique or method learned as an abstraction to a given concrete problem [4, pp. 120–123]. In order for this activity to rank at a higher level than comprehension, the learner should be able to discern independently which abstraction, among several, is suitable for the given problem, and not be instructed to use a specific one.

Analysis, typically considered a critical thinking task, may be viewed as one of the higher-level objectives. It mainly involves the process of decomposing given information into its elementary parts in order to study and understand it [4, pp. 144–145].

Another higher-level objective, Synthesis, relates to tasks that require creativity and original thinking. More explicitly, synthesis is the putting together of parts of known facts, methods, and ideas to create a new whole. The end product of such activity could be a unique piece of communication, an action plan, or a recognized relation between the various components of the problem under study [4, pp. 162–165].

Evaluation is the highest level objective of the cognitive domain in Bloom’s Taxonomy, and refers to tasks that involve making value judgments. Evaluators may be required to apply internal or external criteria in their evaluation [4, pp. 185–187]. As it is the main focus of this article, this level is studied in much more detail in Section 3.

2.2. Calculus Examples for the First Five Levels

Exercises and test questions in most introductory mathematics courses typically address the lower levels of Bloom’s taxonomy. In [9] I present a taxonomic investigation of the exercises from several chapters of a mainstream calculus text, and show that most do not address the higher levels. The scope of this article is more limited. Therefore, in the following, I simply provide a handful of typical calculus problems together with their most likely Bloom classification. As Bloom et al. point out, the task of classifying questions in a
given course is not straightforward [4, pp. 51–54]; various factors need to be considered, such as what the student has been exposed to before being assigned the particular question.

- Identify the local extrema and inflection points on a given graph (Knowledge).
- Define what it means to say that infinite series converges (Knowledge).
- State the Fundamental Theorem of Calculus (Knowledge).
- Explain in your own words what it means to say that an infinite sequence converges (Comprehension).
- Give an example of a definite integral (Comprehension).
- Describe how to use the chain rule to differentiate the function \( f(x) = (\sin x)^2 \) with respect to \( x \) (Comprehension).
- Differentiate the function \( f(x) = (x + 1)^4 \sin(x^4) \) with respect to \( x \) (Application).
- Find the global extrema of the function \( g(x) = x^4 + 16x^2 + 64 \) on the interval \([2, 5]\) (Application).
- Determine the points at which the unique line with positive slope that is tangent to the circles \( x^2 + y^2 = 1 \) and \( y^2 + (x - 3)^2 = 4 \) intersects the two circles (Application).
- Outline your solution to the previous problem, explaining your reasoning for each of the steps (Analysis).
- Point out which properties of limits were applied to evaluate the limit in the (given) solution (Analysis).
- Describe how Rolle’s Theorem was applied in the (given) proof of the Intermediate Value Theorem (Analysis).
- Make a conjecture about how the volume and the surface area of a sphere of radius \( r \) are related to one another by studying the formulas of these quantities. Justify your claim (Synthesis).
- Graph the function \( f(x) = x^3 - x \) as accurately as you can using no graphing calculators (Synthesis).
- Using the rigorous definition of the limit, show that the limit, as \( x \) goes to 5, of \( f(x) = x^2 - 6x + 8 \) is 3 (Synthesis).

3. EVALUATION AND MATHEMATICS

With this section I begin the second and main part of this article. I briefly discuss the sixth level of Bloom’s taxonomy in Section 3.1, and in Section 3.2 I focus on evaluative tasks for the mathematics classroom. In Section 3.3 I describe how these ideas fit in with various philosophical stances on the nature of mathematics and mathematics education.
3.1. Evaluation as an Educational Objective

As mentioned in Section 2.1, Evaluation involves tasks that require one to judge the value of some text, idea, or a proposal. The evaluator is allowed to use internal criteria of the given material, such as consistency or logical accuracy, or can refer to external criteria, such as the purported goal of the evaluated product or standard criteria within a discipline used to evaluate such material. Many times, personal values of the evaluator will influence the final judgment. However, it is crucial that one does not defer to unfounded opinions or instinctive reactions. On the contrary, the end result of evaluative activity needs to be well though out and justified by arguments based on facts and logical deductions.

By their very nature, evaluative tasks can be risky to include in formal educational contexts. Especially in a democratic society, educators typically attempt to withhold their own values and opinions while teaching. The argument is that students should only be exposed to facts and frameworks of ideas; they should then be set free to develop their own opinions and value systems (also see the discussion in [4, p. 188]). Thus, most evaluative tasks students get to engage in during their academic careers involve internal criteria, since these are less open to interpretation within a personal value system. As a result, one may see instructors explicitly listing what criteria students should use in assignments involving evaluative tasks and spelling out in detail what is acceptable as justification. In some instances, this list of criteria becomes so extensive and complete that the student does not have to engage in a truly evaluative process.

3.2. Evaluation and the Mathematics Classroom

Let us now consider evaluative tasks that may be used for mathematics courses. As expected, evaluation tasks that require reference to internal criteria are easier to develop. A type of question that can be used in almost any mathematics class at any grade level is the following:

(Given a purported solution to a problem) grade this solution from a fictitious classmate.

As students learn to compose acceptable solutions to typical problems in a given class, we expect that they also develop a sense of what makes a solution acceptable. Questions of the above type test whether this expectation is justified. In order to answer such questions satisfactorily, students need to refer to criteria such as logical consistency and transparency. If the instructor requires the use of a particular format for solutions, this can be an additional criterion.

A variation of this style of question can be suitable for a class that introduces students to writing proofs:
Indeed, such an activity for abstract algebra students is studied in detail in [10].

Instructors may choose to use questions like the above in homework sets or examinations. Or they may prefer to incorporate such evaluative tasks within student peer review activities. Student peer review is a pedagogical tool used by many instructors (see [8] for an analysis of its effectiveness and a summary of research results about it). Readers of *PRIMUS* may find the recent articles [6, 11] interesting as instances of student peer review within the mathematics classroom.

It is more difficult to find evaluative projects in mathematics that involve external criteria. Here, instructors teaching applicable mathematics are at an advantage, because standard modeling projects may sometimes be modified to include evaluative components. If designed well, such projects allow the mathematics teacher of any grade to engage students at this highest level. For a good example see [12]. In this annotated lesson plan for the middle school mathematics classroom, Renner describes two evaluative projects. In one, students work to determine the best place to put a wheelchair ramp on their school campus. In the other, they are assigned the task of designing a community garden.

In both instances, besides cost, what makes a proposal the “best” interpretation and, hence, evaluation.

Most modern calculus textbooks include some modeling projects. *PRIMUS*, along with many other pedagogical publications, routinely publishes examples of calculus modeling projects; a search on ERIC (Education Resources Information Center: http://www.eric.ed.gov/) within *PRIMUS* for the keywords *calculus* and *modeling* returns 12 different articles. However, a close look at these projects shows that most of them, though open-ended to an extent, are not very conducive to evaluative activities. There is generally a clear understanding of what the students are expected to do or at least what the end result should look like, and so there is no room for value judgments.

### 3.3. Is Mathematics Value-Free?

Briefly ponder the following questions: Is mathematics value-free? What does this question mean? When we make assertions like, “Science is not value-free,” what do we mean? A comprehensive discussion of these questions would require us to analyze objectivist and social constructivist philosophies of mathematics and would definitely lead us out of the scope of this article. Thus, we simply mention a few leads here; we encourage the reader to check out these references and continue the conversation elsewhere.
The dominant objectivist/absolutist approach to mathematics, traditionally assumed within thought paradigms such as logicism, formalism, and platonism, asserts that mathematics is a collection of absolute facts and certain knowledge, and thus does not allow for value judgments. For a contemporary overview and defense of this perspective, see [13]. Social constructivist researchers question this stance, and in particular argue that mathematical knowledge is a contingent social construction (see, for instance, [7]). Yet others argue instead that while it is possible that mathematical truths themselves are objective, the way individuals and societies access mathematical knowledge is laden with subjectivity, including but not limited to, cultural biases, personal errors, and historical idiosyncrasies. A clear account of this discussion from a mathematician thinking about the values of mathematics can be found in [15].

Reading the relevant literature even superficially makes it clear that the argument on the value-free nature of mathematics is not settled. This should not prevent us from engaging our students in evaluative activities in the mathematics classroom. Even if we believe that mathematics is value-free, it can be argued that mathematicians should incorporate a discussion of values and morals into their instruction [14]. My proposal here is much more modest: I merely suggest that mathematics teachers consider developing and using evaluative components in their courses. If one of our goals in our classes is to help our students develop into effective thinkers, then providing appropriate contexts in which they get to practice their decision-making skills is only reasonable, and evaluative assignments can do just that.

4. AN EVALUATIVE CALCULUS PROJECT

4.1. The Project Description

As with most mathematics instructors, I often find myself teaching calculus. At my college, the first semester calculus course, Calculus I, initiates students to both differential and integral calculus. Traditionally, differential calculus is emphasized more, as it is expected that students will subsequently spend a semester on Calculus II, which focuses on integral calculus. This assumption is not valid for all students however; many take Calculus I as a general education requirement, and do not move on to the second course of the sequence. Therefore, when teaching this course, I try to balance the two parts. In particular, while I use a standard text, I introduce integration earlier than most. Furthermore, I focus on applications of both differential and integral calculus throughout the semester.

In the Fall 2008 semester when I taught Calculus I, I used the following capstone project:

The main idea: You are to write an argumentative essay of 1000 words or more about the following situation: You are invited to be a student
member of the curriculum committee of the mathematics department. The committee has been assigned the task of developing a ten-week-long calculus course intended to fulfill a quantitative component for the general education requirement of students from various majors. The big-wigs on the committee have already decided that the course will be either in differential calculus or integral calculus, and certainly not both. You, of course, may have a different opinion, but this decision is made and cannot be altered. Your job is to think about this issue, pick a side (i.e., decide whether to offer the course in differential calculus or in integral calculus), and come up with a coherent argument as to why that should be the final decision of the committee. You can use any plausible argument you can justify (and your justification may be based on historical or pedagogical concerns, or depend on applicability to various majors, etc.).

In a note on the course web page, I shared with my students that my main motivation with this project was to engage them at the evaluative level. I put in a simplified description of Bloom's Taxonomy of the cognitive domain, and provided them with references in case anyone wanted to pursue these ideas. I did not expect that this would be of interest to most students, but I thought that some might conceivably find it intriguing, and, for those, the linked resources could encourage them to reflect deeper upon their experiences.

4.2. Analysis of the Project

While designing this project, I anticipated a diverse selection of student responses. In particular, I expected arguments on both sides, supported by historical, pedagogical, or application-oriented justifications. I believe, in fact, that strong cases can be made on each side.

Most modern texts introduce differential calculus first, and many depend solely on the Fundamental Theorem of Calculus to evaluate integrals. Introduced in this manner, integration seems to naturally follow differentiation, and conceptually depend on it. Thus, one can suggest that integration on its own is not as fundamental, and students in the ten-week general education course should only be taught differential calculus if there needs to be a choice. Many students have internalized the main interpretation of the derivative as a rate of change, by the end of one semester of study, and they can provide many examples of applications of this notion to disciplines ranging from economics to physics to population dynamics. This may lead some students to the choice of differential calculus as the more useful part of calculus.

On the other hand, both the derivative and the integral depend on the notion of limits, and once limits are introduced, the next step can be either the derivative or the integral. One can therefore conceive of a course of study where students are introduced to the notion of limits of infinite series, and then...
integrals. (Indeed, this is the path followed by a long time favorite of many mathematicians; see the table of contents for [3]). Some techniques to add up basic forms of infinite series can be provided. Then several computational examples can follow. Allowing the use of computer technology to approximate infinite sums makes this a feasible enterprise, as students can use the computer for sums they cannot do by hand. Along the way, students can be exposed to the notions of mathematical induction, approximation error, and the intuitive interpretation of the definite integral as an area. The notion of area is more intuitive to most students than that of the tangent line. Thus, one can argue that this path is an acceptable, and in fact desirable, path to follow. Applications of the integral abound in many disciplines, and students who favor such applications may also lean toward arguing for integral calculus.

Historical concerns may also lead students in either direction. If we view integration as a formalized way to compute areas, simply noting that area computations involving limits (e.g., the area of a circle as the limit of polygons with increasing number of sides) can be traced back to thousands of years ago may convince some of the more fundamental nature of the integral. On the other hand, if one decides that the main power of calculus is embedded in the Fundamental Theorem of Calculus, and that before Newton and Leibniz perfected the technique via the use of what we now call differentials, there was no calculus as we know it today, then perhaps this revolutionary nature of the differential calculus may persuade others to choose that side.

Some might argue that a single semester of calculus may not be sufficient to give the students enough depth in the matter to provide for a very multifaceted discussion of the project theme. In fact, I agree that a two-semester sequence could provide a more appropriate background for this project. Then students would have more experience with the two branches of calculus, and could develop a more balanced view. However, due to the general education audience of my class, waiting until the end of the second semester course was not an option.

Others may argue that calculus students are not mature enough to make evaluative assessments of their education, and this project is not appropriate for their levels at all. I disagree with this position. If we do not expect great things from our students, we will never know what they can deliver. Philosophy instructors do not avoid topics like free will and the mind-body problem in their freshman seminars. Of course this puts an onus on the instructor to prepare students well through the semester; I describe how I went about doing this in my class in the following section.

4.3. Notes from the Classroom

At the beginning of the semester I had envisioned using this project as a topic for an in-class debate. I was hoping to start students thinking on the topic early on, continuously collecting thoughts and arguments to favor either side, and
then at the end of the term I would select teams to defend each side. The rest of
the class would be the well-informed audience who gets to decide the winner
of the debate. However, as weeks went by and the schedule straightened itself
out for this specific class, I realized that time-wise this was not going to be
feasible. To be honest about it, I should admit that I also felt unprepared to lead
a classroom debate. Thus, the project became an end-of-term writing project.

Many students were excited with a writing project in their mathematics
class. They took the project seriously, and the end results were almost uni-
formly well written. Overall, I was impressed that almost all students faced the
challenge head-on and produced high-quality work.

I ended up with a majority of students choosing differential calculus over
integral calculus. Nonetheless, I did have a sufficient number of integration
supporters to become convinced that this project turned out to be at least par-
tially successful. I have a few hypotheses as to why there was not a more even
division of opinions.

As I mentioned above, I planned the course schedule so that the amount
of time we spent on integration would be more or less balancing the amount
spent on differential calculus. In particular, I managed to spend equal time on
the definitions and the applications of the derivative and the integral. However,
since the follow-up course, Calculus II, focuses on techniques of integration, I
was required to teach techniques of differentiation in my class, so students still
got more exposure to the idea of differentiation. Furthermore, many students
had already taken some form of calculus in high school, and that also seemed
to have ingrained in them the notion that “you begin with differential calculus
and that is the way to go.”

Nonetheless, there were quite a few eager supporters of integral calculus
among my students. Many mentioned favored applications, and some focused
on the ease of comprehending the notion of area as opposed to the notion that
(the slope of) the tangent line represents a rate of change.

Not too many students chose to refer to historical precedence. I think
that as I tweak my syllabus to include more historical tidbits, some of that
will perhaps seep into the arguments used. Also, the computability of inte-
grals by iterated sums did not appear to play a role in my students’ decisions.
Emphasizing that may be a good idea the next time I teach the course. I also
plan on putting even greater effort into specifically motivating integral calculus,
independent from differential calculus.

After the course ended, a few students told me they were disappointed
that the end-of-the-semester debate did not take place; they had been looking
forward to it. I was glad that they shared their opinions on this, because I had
not been really sure of the suitability of using the debate format within the
mathematics classroom. I will be considering this more seriously the next time.
In particular, I plan to consult colleagues from disciplines where student debate
is a standard classroom technique, and get some pointers. I also hope to find
the time to have my students watch the infamous π/e debate [1] to get them to
think about what a mathematical debate might look like.
When I designed the project described here, I was mainly looking to find an interesting way to create an evaluative activity for my students. However, I believe that the project also provided my students an opportunity to think critically about why they themselves were taking a calculus course in the first place, and what, if anything, they were getting out of it. Overall, the particular task of writing up their contributions for the project as coherently as possible created a more reflective, more conscious, and thus a more effective learning experience for all involved.

5. CONCLUSION

As mathematics instructors we have many decisions to make when developing our courses. In this article I argued that, among other things, we should consider incorporating evaluative components into our courses. As an example, I introduced one such project, which I have used in one of my courses. Obviously, my experience is limited and thus my observations are tentative. Nonetheless, I found it to be a most eyeopening experience, and, in particular, I am convinced more than ever that evaluative tasks have a place in all mathematics classrooms.

I would be happy to continue this discussion with interested readers. I especially encourage them to share their general thoughts about evaluation in the mathematics classroom, and, more specifically, evaluative projects that they may have developed for their own classes. A collection of evaluative tasks of varying size and complexity would be a welcome addition to our pedagogical toolboxes.

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REFERENCES


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